

Strings on Eight-Orbifolds

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Abstract

We present several examples of T^8/P orbifolds with $P \subset SU_4$. We compute their Hodge numbers and consider turning on discrete torsion. We then study supersymmetric compactifications of type II, heterotic, and type I strings on these orbifolds. Heterotic compactifications to $D=2$ have a B -field tadpole with coefficient given by that of the anomaly polynomial. In the SO_{32} heterotic with standard embedding the tadpole is absent provided the internal space has a precise value of the Euler number. Guided by their relation to type I, we find tadpole-free SO_{32} heterotic orbifolds with non-standard embedding.

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1 Introduction

F-theory compactified on certain Calabi-Yau four-folds leads to $\mathcal{N}=1$ four-dimensional vacua that can have phenomenological applications [1]. Compactification of M-theory and type IIA strings on the same manifolds gives related lower-dimensional vacua that have some interesting features of their own [2, 3, 4, 5, 6]. In this note we study T^8/P orbifolds, with $P \subset SU_4$, that share many properties of the smooth spaces and allow a simpler computation of the string spectra and interactions [7].

We consider Abelian point groups P that can be products of up to three cyclic factors. We classify all crystallographic \mathbb{Z}_N actions and also present examples with more factors in which discrete torsion is allowed. The Hodge numbers $h_{p,q}$ of these orbifolds are efficiently computed using their connection to superconformal field theories. We exclude P 's that leave a sub-torus invariant, so that $h_{0,1} = h_{0,3} = 0$ but in general $h_{0,2} \neq 0$. In section 2 we review some basic properties of Kähler four-folds with vanishing first Chern class and collect some useful results that also apply to our orbifolds.

Examples of compactification on T^8/P orbifolds have been described by several authors [8, 9, 10, 11, 12, 13]. In section 3 we explicitly compute the massless spectra of type II orbifolds and show that they coincide sector by sector with known results obtained from Kaluza-Klein reduction [5, 14, 15, 16]. We review this reduction in order to include the case $h_{0,2} \neq 0$ that signals extended supersymmetry.

Heterotic strings generically have a B -field tadpole in $D=2$ [2]. To examine this problem we first consider compactification on smooth Calabi-Yau four-folds with standard embedding. The full charged spectrum follows from analyzing the zero modes of $D=10$ gaugini whereas the number of net gauge singlet spinors can be determined from anomaly factorization. We verify equality of the coefficient of the $D=2$ anomaly polynomial and the tadpole obtained by integration of the anomaly canceling term in $D=10$, as expected on general grounds [17]. The tadpole can only be canceled in the SO_{32} heterotic provided the four-fold has Euler number $\chi = 180$. Analogous results hold in orbifolds with standard embedding. When $h_{0,2} \neq 0$ tadpole cancellation requires $\chi = 90(2 + h_{0,2})$.

We are naturally led to look into heterotic orbifolds with general modular invariant embeddings. We indeed find SO_{32} models in which the full anomaly and thus the tadpole

vanishes. These examples are closely related to type I strings that are our last topic. We discuss in some detail $D=2$ type I vacua realized as type IIB orientifolds. Some models of this kind were constructed in [18]. While these theories are necessarily free of gravitational and gauge non-Abelian anomalies, they can have U_1 anomalies that are shown to be canceled by exchange of RR scalars.

Section 4 is devoted to concluding remarks. The data of the T^8/P orbifolds is collected in four tables found at the end of the paper.

2 Eightfolds

2.1 Calabi-Yau

Calabi-Yau d -folds (CY_d 's) are of special interest because they admit a reduced number of covariantly constant spinors. For a CY_d the holonomy group is strictly SU_d . In this section we collect some useful facts about CY_4 's that are characterized by Hodge numbers $h_{p,q}$ satisfying $h_{p,q} = h_{q,p}$, $h_{p,q} = h_{4-p,4-q}$ and $h_{0,p} = h_{4-p,0}$. Furthermore, $h_{0,0} = h_{0,4} = 1$. Recall that the Betti numbers are $b_n = \sum_{p+q=n} h_{p,q}$.

In general, a complex Kähler 4-fold Y with vanishing first Chern class, $c_1[Y] = 0$, has holonomy inside SU_4 . We assume that the manifold is neither T^8 nor products $T^2 \times CY_3$, $T^4 \times K3$ so that $h_{0,1} = h_{0,3} = 0$. The independent Hodge numbers are $h_{0,2}$, $h_{1,1}$, $h_{1,2}$ and $h_{1,3}$. Using results given in [19] one can show that

$$h_{2,2} = 2(22 + 10h_{0,2} + 2h_{1,1} + 2h_{1,3} - h_{1,2}) . \quad (1)$$

The Euler characteristic can thus be written as

$$\chi = 6(8 + 4h_{0,2} + h_{1,1} + h_{1,3} - h_{1,2}) . \quad (2)$$

Notice that χ is always multiple of six. When $h_{0,2}$ also vanishes the manifold is a CY_4 . Another quantity of interest is the signature $\tau = b_4^+ - b_4^-$, where b_4^+ and b_4^- are the number of self and antiself-dual harmonic 4-forms. Clearly, $b_4^+ + b_4^- = 2 + 2h_{1,3} + h_{2,2}$. From the expression of τ in terms of Chern classes [19] we find

$$\begin{aligned} b_4^- &= 2h_{1,3} + h_{1,1} - 2h_{0,2} - 1 \\ b_4^+ &= 47 + 22h_{0,2} + 4h_{1,3} + 3h_{1,1} - 2h_{1,2} . \end{aligned} \quad (3)$$

In section 3.2 we will recover these results from supersymmetry and anomaly cancellation in type IIB compactification on the four-fold. Notice that $\tau = \frac{\chi}{3} + 32 + 16h_{0,2}$.

Other useful results derived from general properties of four-folds with $c_1[Y] = 0$ are

$$\begin{aligned} \frac{1}{5760(2\pi)^4} \int_Y \text{tr } R^4 + \frac{5}{4}(\text{tr } R^2)^2 &= 2 + h_{0,2} \\ \frac{1}{8(2\pi)^4} \int_Y -\text{tr } R^4 + \frac{1}{4}(\text{tr } R^2)^2 &= \chi, \end{aligned} \quad (4)$$

where R is the curvature 2-form.

There is a family of CY₄'s with structure K3 × K3/ σ where the involution σ reverses the sign of the (2,0) forms of each K3 but leaves the (4,0) form of the 4-fold invariant [20]. The action of σ on each K3 is characterized by two 3-plies (r_i, a_i, δ_i) , $i = 1, 2$ and the Hodge numbers turn out to be

$$h_{1,1} = r_1 + r_2 + f_1 f_2 \quad ; \quad h_{1,2} = f_1 g_2 + g_1 f_2 \quad ; \quad h_{1,3} = 40 - r_1 - r_2 + g_1 g_2 , \quad (5)$$

where $f_i = 1 + (r_i - a_i)/2$ and $g_i = 11 - (r_i + a_i)/2$. This is a generalization of the class of Voisin-Borcea CY₃'s with structure T² × K3/ $\hat{\sigma}$, where the involution $\hat{\sigma}$ reverses the signs of the T² (1,0) form and the K3 (2,0) form. Since the K3's can be obtained by modding T⁴ by a \mathbb{Z}_M action we expect to find orbifolds whose Hodge numbers coincide with (5). Clearly we can also think of 4-folds that are elliptic fibrations of the form T² × CY₃/ $\tilde{\sigma}$ where $\tilde{\sigma}$ reverses the signs of the T² (1,0) and the CY₃ (3,0) forms.

2.2 Orbifolds

For an orbifold T^{2d}/P, the point group P is the holonomy group. We consider Abelian point groups P given by products of \mathbb{Z}_N factors, with each factor generated by a rotation θ that acts on the T^{2d} complex coordinates as

$$\theta X_i = e^{2i\pi v_i} X_i \quad , \quad i = 1, \dots, d. \quad (6)$$

Clearly $v_i = \text{int}/N$ for θ of order N. Existence of covariantly constant spinors, that will give unbroken supersymmetries upon string compactification on the orbifold, requires

$$\sum_i \pm v_i = 0 \bmod 2, \quad (7)$$

for some choice of signs. This is the condition $P \subset SU_4$. For definiteness we choose $\sum_i v_i = 0$ and also demand that no sub-torus be left invariant. Then, there are at least two solutions of (7). If there are no extra solutions, the orbifold is a singular limit of a CY_d manifold. Extra solutions could appear for $d \geq 4$, in particular for $d = 4$ they actually imply $h_{0,2} > 0$.

There is also a condition that P acts crystallographically on the torus lattice Λ . The allowed Abelian actions can be found combining proper \mathbb{Z}_N twists as explained in [21]. In eight dimensions such proper twists exist for $N = 15, 16, 20, 24, 30$ and have exponents a_i/N , where $a_i < N/2$ are integers relative prime to N . In all cases, we can find supersymmetric twists. Except for $N = 16$ they can be realized on the E_8 root lattice by elements of the Weyl group, the \mathbb{Z}_{30} being the Coxeter rotation. The \mathbb{Z}_{16} rotation is the generalization of the \mathbb{Z}_4 rotation acting on the SO_4 lattice, it can be simply realized on a hypercubic lattice with orthonormal basis.

Combining the lower dimensional proper twists given in Table 1 of [21] it is a simple exercise to find all other possible crystallographic inequivalent supersymmetric actions. For example, for \mathbb{Z}_N obtained in this way there is one solution for each $N = 2, 3, 5, 9, 14, 18$, and there are respectively 3, 7, 4, 2, 10, 2, for $N = 4, 6, 8, 10, 12, 24$. Whenever allowed we take the torus lattice Λ to be the product of two-dimensional SO_4 or SU_3 root lattices. Some exceptions are the \mathbb{Z}_5 , \mathbb{Z}_8 , \mathbb{Z}_9 and \mathbb{Z}_{14} for which Λ can be taken to be respectively the product of two SU_5 root lattices, the hypercubic lattice, the SU_9 root lattice, and the SO_{16} root lattice. In many cases one could also use the E_8 lattice. The inequivalent sets of v_i 's are either of type $(\frac{1}{a}, -\frac{1}{a}, \frac{1}{b}, -\frac{1}{b})$, $a, b = 2, 3, 4, 6$; $\frac{1}{N}(1, 1, 1, -3)$, $N = 4, 6$; $\frac{1}{N}(1, 3, -2, -2)$, $N = 6, 8$, $\frac{1}{N}(1, 3, -1, -3)$, $N = 5, 8, 10$; or are in the following list

$$\begin{array}{ccccc} \frac{1}{6}(1, 1, 2, -4) & \frac{1}{12}(1, 5, -1, -5) & \frac{1}{12}(1, 5, -3, -3) & \frac{1}{15}(1, 2, 4, -7) & \frac{1}{24}(1, 5, 7, -13) \\ \frac{1}{8}(1, 5, -2, -4) & \frac{1}{12}(1, 5, 1, -7) & \frac{1}{12}(1, -5, -4, 8) & \frac{1}{16}(1, 3, 5, -9) & \frac{1}{24}(3, 9, -4, -8) \\ \frac{1}{8}(1, 3, 1, -5) & \frac{1}{12}(1, -5, -2, 6) & \frac{1}{12}(2, 4, -3, -3) & \frac{1}{18}(1, 7, -5, -3) & \frac{1}{24}(3, 9, -2, -10) \\ \frac{1}{9}(1, 4, -2, -3) & \frac{1}{12}(1, 5, -2, -4) & \frac{1}{14}(1, -3, -5, 7) & \frac{1}{20}(1, 3, 7, -11) & \frac{1}{30}(1, -7, -11, 17) \\ \frac{1}{10}(1, -3, -2, 4) & \frac{1}{12}(1, -5, 2, 2). & & & \end{array}$$

Note that the irreducible twists are included.

Let us now briefly explain a simple procedure to compute the cohomology of these toroidal orbifolds. To begin consider a (2,2) superconformal theory with $c = 3d$ realized

by d free complex bosons X_i and fermions ψ_i . It is well known that the ground states of the Ramond-Ramond (RR) sector are in one to one correspondence with the cohomology classes of T^{2d} . Indeed, the Hodge numbers $h_{p,q}$ are given by the number of RR massless states with (p, q) U_1 charges [22].

Next we quotient the theory by P whose elements transform the fermions ψ_i just as the bosons X_i , for instance as (6) for \mathbb{Z}_N actions, so as to preserve the superconformal invariance. As usual, the partition function will include a sum over twisted sectors plus the orbifold projection. Schematically,

$$\mathcal{Z} = \frac{1}{|P|} \sum_{g \in P} \sum_{h \in P} \epsilon(g, h) \mathcal{Z}(g, h). \quad (8)$$

The discrete torsion $\epsilon(g, h)$ satisfies $\epsilon(g, g) = 1$ and furthermore [23]

$$\epsilon(g, h_1 h_2) = \epsilon(g, h_1) \epsilon(g, h_2) \quad ; \quad \epsilon(g, h) \epsilon(h, g) = 1. \quad (9)$$

For instance, when there are several \mathbb{Z}_N factors there is the possibility of non-trivial discrete torsion. For $\mathbb{Z}_N \times \mathbb{Z}_M$ generated by α ($\alpha^N = 1$) and β ($\beta^M = 1$)

$$\epsilon(\alpha^k \beta^\ell, \alpha^s \beta^t) = \epsilon_1^{kt - \ell s}, \quad (10)$$

where $\epsilon_1^{gcd(N,M)} = 1$. Similarly, for the product of three factors with a third generator γ ($\gamma^K = 1$) we have

$$\epsilon(\alpha^k \beta^\ell \gamma^m, \alpha^s \beta^t \gamma^n) = \epsilon_1^{kt - \ell s} \epsilon_2^{\ell u - mt} \epsilon_3^{ku - ms}, \quad (11)$$

where $\epsilon_2^{gcd(M,K)} = 1$ and $\epsilon_3^{gcd(N,K)} = 1$.

From the partition function we can extract the spectrum of states. In particular, for right-movers in the θ -twisted sector the mass formula is

$$m^2(\theta) = \frac{1}{2}(r + v)^2 + N_R + E_0 - \frac{c}{24}, \quad (12)$$

where r is an SO_{2d} weight that arises from bosonization of the fermions and the twist vector v has components v_i . N_R is an oscillator number for all bosons and E_0 is the shift in vacuum energy given by

$$E_0 = \sum_i \frac{1}{2} |v_i|(1 - |v_i|). \quad (13)$$

To each state we can associate the U_1 charge

$$q(r) = \sum_i r_i + \frac{c}{6}. \quad (14)$$

Recall that r is an spinorial weight in the R sector. For left-movers there are analogous results. The number of RR massless states with (p, q) U_1 charges is the Hodge number $h_{p,q}$ of T^{2d}/P . Since we have chosen $\sum_i v_i = 0$, it is easy to see that there is only one state, appearing in the untwisted sector, with charges $(0, d)$. Hence, $h_{0,d} = 1$ as it should for a space of holonomy inside SU_d .

To find the degeneracy of the RR ground states with (p, q) charges we simply use the orbifold projector obtained from the partition function [24]. In Tables 1, 2, 3 and 4 we present the Hodge numbers for a few selected orbifolds. Some comments are in order.

- All \mathbb{Z}_N orbifolds with proper twist in eight dimensions, thus with $N = 15, 16, 20, 24, 30$, have the same Hodge numbers as the \mathbb{Z}_{15} displayed in Table 1.
- The first entry in Table 2 corresponds to orbifold limits of $K3 \times K3$.
- In Table 2, the $\mathbb{Z}_2 \times \mathbb{Z}_3$ and the $\mathbb{Z}_2 \times \mathbb{Z}_7$ are actually \mathbb{Z}_6 and \mathbb{Z}_{14} . We choose to give the product realization because it shows the structure $T^2 \times CY_3/\tilde{\sigma}$. Similarly, some examples in Table 3 can be seen as $\mathbb{Z}_2 \times \mathbb{Z}_6$ or $\mathbb{Z}_3 \times \mathbb{Z}_6$.
- The first 3 examples in Table 3 with $P = \mathbb{Z}_2 \times \mathbb{Z}_M \times \mathbb{Z}_2$ have the Borcea structure $K3 \times K3/\sigma$, with σ given by the second \mathbb{Z}_2 . Indeed, the Hodge numbers coincide with (5) upon taking $r_1 = 18, a_1 = 4$ together with $r_2 = 18, 14, 16, a_2 = 4, 6, 8$ for $M = 2, 3, 4$. These models can also be regarded as elliptic fibrations $T^2 \times CY_3/\tilde{\sigma}$, with $\tilde{\sigma}$ given by the first \mathbb{Z}_2 . Moreover, the CY_3 is of Voisin-Borcea type $T^2 \times K3/\hat{\sigma}$ and the r_2, a_2 are precisely those that reproduce the CY_3 Hodge numbers when the $K3$ is realized as T^4/\mathbb{Z}_M [20, 25]. The example with $P = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ in Table 3 has Hodge numbers that agree with (5) for $r_1 = 18, a_1 = 4, r_2 = 10$ and $a_2 = 8$.
- It is easy to show that in all T^8/P Abelian orbifolds $h_{0,2}$ is even as observed in the Tables. However, there is no such constraint on $h_{0,2}$ in generic Kähler four-folds with vanishing first Chern class. For example, there is a smooth, simply-connected, manifold with $h_{0,2} = 1, h_{1,1} = h_{1,3} = 21, h_{1,2} = 0$ and holonomy Sp_2 . This manifold is obtained by

taking the quotient of $K3 \times K3$ by the involution that exchanges both factors and then blowing-up [26].

- In Table 4 we collect some representative orbifolds with discrete torsion. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ was described in detail in [4] as an example with negative Euler characteristic. As noticed in [4], χ can become negative because discrete torsion tends to increase the value of $h_{1,2}$. The two $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ were discussed in [12] where it was shown that the mirror of the orbifold without discrete torsion is obtained taking $\epsilon_2 = \epsilon_3 = -1$. For a mirror pair Y , Y^* , $h_{p,q}(Y) = h_{4-p,q}(Y^*)$.
- All examples have χ multiple of 24. The Euler characteristic can be computed from the Hodge numbers or from the master orbifold formula of [7]. Likewise, $h_{2,2}$ can be found using (1) or directly from the number of RR states with (2,2) charge.
- To properly discuss the resolution of singularities in these orbifolds is beyond the scope of this paper. In the T^8/\mathbb{Z}_N with twist vector of the form $\frac{1}{N}(1, -1, 1, -1)$, $N = 2, 3, 4, 6$, or $\frac{1}{N}(1, 3, -1, -3)$, $N = 5, 8, 10$, the singularities are known to be of terminal type [27]. A property shared by orbifolds with terminal singularities is that only the untwisted sector contributes to $h_{1,1}$. For the rest of the \mathbb{Z}_N actions there is at least one twisted sector θ^k (and its inverse θ^{N-k}) that adds to $h_{1,1}$. The condition for the existence of such sectors, when written in terms of the corresponding twist with components kv_i , coincides with the criterion that prevents the singularity from being terminal [28, 29]. Borrowing the notation of [29], the condition is $\sum_i \langle kv_i \rangle \leq 1$, where $\langle kv_i \rangle \in [0, 1]$ is obtained from kv_i by adding or subtracting one.

3 Compactification

To compute the massless fields of type II or heterotic strings compactified on a Calabi-Yau four-fold we can consider the ten-dimensional bosonic fields whose zero modes are easy to count. Massless fermion fields follow via supersymmetry but it is instructive, in particular to reduce charged fields, to determine the fermionic zero modes. To this end we use the same procedure applied in the case of three-folds [30, 31].

Let ψ^c be an $SO_{1,9}$ Weyl spinor in the **16**_c. We can write the $SO_{1,1} \times SO_8$ decomposition

$$\psi^c = \psi^+ \otimes \eta^s + \psi^- \otimes \eta^c , \quad (15)$$

where ψ^\pm are $D=2$ Weyl spinors of \pm chirality, whereas η^s and η^c are SO_8 Weyl spinors. In terms of spinorial weights $(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2})$, η^s has even and η^c odd number of +'s, notice that each spinor is its own complex conjugate. When ψ^c is Majorana-Weyl, so are ψ^\pm . A spinor ψ^s in the **16**_s of $SO_{1,9}$ has a decomposition analogous to (15) with ψ^+ and ψ^- exchanged.

In a Kähler four-fold of holonomy inside SU_4 , spinors can be written in terms of the covariantly constant spinors and Dirac matrices that act as creation operators. Concretely,

$$\eta^s = \alpha \eta_+ + \alpha_{i\bar{j}} \gamma^i \gamma^{\bar{j}} \eta_+ + \alpha_{\bar{i}j\bar{k}\bar{l}} \gamma^{\bar{i}} \gamma^j \gamma^{\bar{k}} \gamma^{\bar{l}} \eta_+ . \quad (16)$$

The indices $i, \bar{i} = 1, \dots, 4$, refer to the local complex coordinates. The Dirac algebra is $\{\gamma^i, \gamma^j\} = \{\gamma^{\bar{i}}, \gamma^{\bar{j}}\} = 0$, $\{\gamma^i, \gamma^{\bar{j}}\} = 2g^{i\bar{j}}$, where $g^{i\bar{j}}$ is the Kähler metric. The spinors η_+ and $\eta_- = \gamma^{\bar{1}} \gamma^{\bar{2}} \gamma^{\bar{3}} \gamma^{\bar{4}} \eta_+$ are covariantly constant, with η_+ characterized by $\gamma^j \eta_+ = 0$. The decomposition (16) reflects the branching of the **8**_s of SO_8 into **1+6+1** of SU_4 . Similarly,

$$\eta^c = \alpha_{\bar{i}} \gamma^{\bar{i}} \eta_+ + \alpha_{\bar{i}\bar{j}\bar{k}\bar{l}} \gamma^{\bar{i}} \gamma^{\bar{j}} \gamma^{\bar{k}} \gamma^{\bar{l}} \eta_+ , \quad (17)$$

which corresponds to $\mathbf{8}_c = \mathbf{4} + \bar{\mathbf{4}}$.

The crucial fact in the expansions (16) and (17) is that the coefficients $\alpha_{i\bar{j}\dots}$ are $(0, q)$ forms. Moreover, zero modes of the Dirac operator are given by spinors whose coefficients are harmonic forms. Thus, in a CY₄ manifold, in which $h_{0,0} = h_{0,4} = 1$ and other $h_{0,q} = 0$, there are no zero modes from η^c and two zero modes from η^s , namely the SU_4 singlets η_+ and η_- . From the expansion (15) it follows that a massless dilatino ψ^c in $D=10$ gives rise to two massless ψ^+ 's in $D=2$. We will also allow for $h_{0,2} \neq 0$ in which case the number of ψ^+ 's from ψ^c is \mathcal{N} , where

$$\mathcal{N} = 2 + h_{0,2} . \quad (18)$$

The space-time components ψ_μ^c of a $D=10$ Majorana-Weyl (M-W) gravitino ψ_M^c also have an expansion of type (15), with $D=2$ spinors ψ_μ^\pm . Hence, the zero modes of η^s give \mathcal{N} M-W gravitini ψ_μ^+ of positive chirality in $D=2$. Similarly, a $D=10$ gravitino ψ_M^s of opposite chirality gives \mathcal{N} gravitini ψ_μ^- of negative chirality in $D=2$.

Expansions of the form (16) and (17) also apply to spinors with an extra holomorphic index. Both η_i^s and η_i^c have coefficients that are $(1, q)$ forms and the spinors are zero modes of the Dirac operator iff the forms are harmonic. For the complex conjugates $\eta_{\bar{i}}^s$ and $\eta_{\bar{i}}^c$ the coefficients are $(q, 1)$ forms. Thus, for example, the zero modes of ψ_i^c and $\psi_{\bar{i}}^c$ give rise to $2h_{1,2}$ massless $D=2$ M-W spinors of positive and $2(h_{1,1} + h_{1,3})$ of negative chirality. From ψ_i^s and $\psi_{\bar{i}}^s$ we find an analogous result exchanging chirality.

Zero modes of bosonic fields are counted as usual. An n -form field gives b_n massless scalars. Recall that in $D=2$ modes with space-time indices have no degrees of freedom on-shell. The metric G_{MN} gives the graviton and real scalars, $h_{1,1}$ from $G_{i\bar{j}}$ and $2(h_{1,3} - h_{0,2})$ from G_{ij} and $G_{\bar{i}\bar{j}}$. The need to subtract $2h_{0,2}$ is a general result [34], it can be simply seen in counting the number of scalars arising from the metric in a T^{2d} compactification.

We use conventions such that $D=2$ massless Weyl fermions of positive chirality are left-moving. Also, a left-moving massless scalar corresponds to a self-dual 1-form. With these results we turn below to determining the massless spectrum of type II, heterotic and type I compactifications. Type II compactification on smooth Calabi-Yau four-folds is well known [5, 14, 15, 16]. We will repeat the analysis in order to allow for the case $h_{0,2} \neq 0$ that is common in orbifolds.

3.1 Type IIA

In type IIA the two $D=10$ gravitini have opposite chirality. Our previous discussion shows that there is an equal number \mathcal{N} of positive and negative chirality gravitini in $D=2$. Hence, the resulting theory has $(\mathcal{N}, \mathcal{N})$ supersymmetry. The gravitini components with internal indices produce equal number $2(h_{1,1} + h_{1,3} + h_{1,2})$ of positive and negative chirality Majorana-Weyl ‘modulini’ in $D=2$. The supersymmetric partners arise from the bosonic fields. There are $(h_{1,1} + 2h_{1,3} - 2h_{0,2})$ real scalars from the metric G_{MN} , $(h_{1,1} + 2h_{0,2})$ from the NS-NS 2-form B_{MN} and $2h_{1,2}$ from the R-R 3-form C_{MNP} . There are no dynamical modes from the R-R 1-form. The dilaton gives one scalar that belongs in the gravity multiplet together with the \mathcal{N} dilatini of positive and negative chirality and the non-dynamical metric and gravitini.

The multiplets of $(\mathcal{N}, \mathcal{N})$ supersymmetry in $D=2$ can be obtained by dimensional

reduction of the multiplets of $D=4$ supersymmetry with $2\mathcal{N}$ supercharges. The $D=4$ chiral multiplet with \mathcal{N} real scalars and $\mathcal{N}/2$ Majorana spinors gives a $D=2$ chiral multiplet with \mathcal{N} real scalars and equal number \mathcal{N} of positive and negative chirality M-W spinors. For $\mathcal{N} = 2, 4, 8$ the $D=4$ vector multiplet has one vector, $(\mathcal{N} - 2)$ scalars and $\mathcal{N}/2$ Majorana spinors, for $\mathcal{N} = 6$ the content is the same as for $\mathcal{N} = 8$. Reducing the vector multiplet gives a vector multiplet with the same content in a chiral multiplet plus a non-dynamical vector.

Given the fermionic and bosonic zero modes explained above, we see that for $\mathcal{N} = 2, 4, 8$ there are $2(h_{1,2} + h_{1,3})/\mathcal{N}$ chiral multiplets and $2h_{11}/\mathcal{N}$ vector multiplets (with non-dynamical vectors arising from $C_{\mu i\bar{j}}$). For $\mathcal{N} = 6$ the multiplets are the same as for $\mathcal{N} = 8$.

Up to now we have neglected the effect of the B -field tadpole that appears generically in IIA compactifications to $D=2$ [2, 3, 4]. This tadpole can be canceled by introducing a number n of fundamental strings given by

$$n = \frac{\chi}{24} - \frac{1}{8\pi^2} \int_Y dC \wedge dC . \quad (19)$$

When $\chi/24$ is a positive integer the tadpole can be canceled, without breaking supersymmetry and without fluxes, just by adding this number of fundamental strings. Each string has a matter content given by the light-cone world-sheet fields, i.e. eight real scalars, eight positive and eight negative chirality M-W fermions. Then, for example, for $\mathcal{N} = 2, 4, 8$, there will be $\chi/3\mathcal{N}$ further chiral multiplets.

Type IIA compactification on orbifolds can be carried out explicitly. The case of T^8/\mathbb{Z}_2 was first presented in [9]. For other orbifolds the massless states can be found using the standard construction, explained for instance in the appendix of [32]. As in [9], in many g -twisted sectors there are no RR massless scalars because the GSO projection forbids massless R states either for left-movers or right-movers. The exception is when g leaves some direction unrotated, e.g. the untwisted sector or the β sector in the $\mathbb{Z}_N \times \mathbb{Z}_M$ examples in Table 2. In such sectors the RR states could still be eliminated by the orbifold projection.

As it should, the number of massless chiral and vector multiplets in IIA orbifolds agrees with the general analysis for compactification on a smooth manifold, with the Hodge

numbers computed in the orbifold sense as explained in section 2.2. The agreement is sector by sector. As an example, take the \mathbb{Z}_3 in Table 1. Together with $h_{2,2}^g$ and χ^g that will be needed in Type IIB and heterotic compactifications, the relevant Hodge numbers in sectors twisted by $g = \theta^n$ are

g	$h_{1,1}^g$	$h_{1,2}^g$	$h_{1,3}^g$	$h_{2,2}^g$	χ^g	
θ^0	8	4	8	18	54	.
$\theta + \theta^2$	0	0	0	162	162	

(20)

In type IIA on T^8/\mathbb{Z}_3 , with $h_{0,2} = 4$ and $\mathcal{N} = 6$, we find 3 chiral and 2 vector multiplets in the untwisted sector, and no multiplets in the twisted sectors. As another example, take the \mathbb{Z}_6 in Table 1, with $h_{0,2} = 0$, $\mathcal{N} = 2$ and

g	$h_{1,1}^g$	$h_{1,2}^g$	$h_{1,3}^g$	$h_{2,2}^g$	χ^g	
θ^0	8	2	0	18	30	.
$\theta + \theta^5$	9	0	0	0	18	.
$\theta^2 + \theta^4$	0	0	0	90	90	
θ^3	6	10	5	24	6	

(21)

Correspondingly, we find 2 chiral and 8 vector multiplets in the untwisted sector, 9 vector multiplets in $\theta + \theta^5$, no multiplets in $\theta^2 + \theta^4$, and 15 chiral plus 6 vector in θ^3 .

3.2 Type IIB

In type IIB the two $D=10$ gravitini have the same chirality, say **16_c**, so that they produce $2\mathcal{N}$ positive chirality gravitini in $D=2$. Hence, the resulting theory has $(2\mathcal{N}, 0)$ supersymmetry. The gravitini components with internal indices produce M-W spinors, $N(\psi^-) = 4(h_{1,1} + h_{1,3})$ of negative and $N(\psi^+) = 4h_{1,2}$ of positive chirality. The dilaton and the axion give one scalar each, from the metric and the two antisymmetric tensors there are $(3h_{1,1} + 2h_{1,3} + 2h_{0,2})$ scalars. From the R-R 4-form C_{MNPQ} with self-dual field strength, there arise $h_{1,2}$ non-dynamical vectors from components $C_{\mu i j \bar{k}}$ and $C_{\mu \bar{i} j k}$, and massless scalars from components with four internal indices. The self-dual harmonic 4-forms give scalars that are left-moving or positive chirality. Hence, there are b_4^+ scalars of positive and b_4^- of negative chirality.

Excluding the dilaton that belongs to the gravity multiplet, the total number of positive and negative chirality scalars are

$$\begin{aligned} N(\varphi^+) &= 3h_{1,1} + 2h_{1,3} + 2h_{0,2} + 1 + b_4^+ \\ N(\varphi^-) &= 3h_{1,1} + 2h_{1,3} + 2h_{0,2} + 1 + b_4^- . \end{aligned} \quad (22)$$

By supersymmetry $N(\varphi^-) = N(\psi^-)$ and therefore b_4^- must be given by (3). Also, using the formula for b_4^+ readily gives $N(\varphi^+) = \chi + 4h_{1,2}$.

We now discuss anomaly cancellation as in [5]. Using the conventions of [30], the contributions of ψ_μ^+ , ψ^+ and φ^+ to the anomaly polynomial in $D=2$ are respectively

$$I_{3/2}^+ = -\frac{23}{48}\text{tr } R^2 , \quad I_{1/2}^+ = \frac{1}{48}\text{tr } R^2 , \quad I_0^+ = \frac{1}{24}\text{tr } R^2 . \quad (23)$$

The gravity multiplet that includes $2\mathcal{N}(\psi_\mu^+, \psi^-)$ contributes $I_{grav} = -\mathcal{N}\text{tr } R^2$. Then, the total anomaly is

$$\frac{1}{24}\text{tr } R^2 \left[-24\mathcal{N} + 2h_{1,2} - 2(h_{1,3} + h_{1,1}) + b_4^+ - b_4^- \right] , \quad (24)$$

which cancels by virtue of (3).

Compactification of type IIB on T^8/P orbifolds can also be carried out in detail [32]. However, since the theory is chiral, care has to be taken because in the light-cone space-time is not directly visible. For NS states this is not a problem since, for instance, basically only $\psi_{-\frac{1}{2}}^m |0\rangle$ ($r = (\pm 1, 0, 0, 0)$ in the notation of [32]) gives rise to massless dynamical degrees of freedom in NSNS or NSR sectors. For both right and left-moving R states we must instead relax the GSO projection in a way that a decomposition such as (15) for $D=10$ spinors be manifest. This means that R states in the light-cone can be *both* of type η^s (i.e. r with $\sum_a r_a = \text{even}$ in the notation of [32]) or type η^c ($\sum_a r_a = \text{odd}$). For a **16**_c M-W the former corresponds to positive and the latter to negative chirality $D=2$ M-W spinors. This modification is also necessary for the R sector in heterotic orbifolds and in open strings in orientifolds. In type IIB when we combine right and left-moving R states, the tensor product $\psi^+ \otimes \psi^-$ does not give physical fields (it would be part of a vector) but $\psi^+ \otimes \psi^+$ and $\psi^- \otimes \psi^-$ give respectively positive and negative chirality scalars.

As it should, the type IIB orbifold spectra agree with the general analysis for smooth manifolds. In each sector twisted by g we find $N^g(\varphi^-) = N^g(\psi^-) = 4(h_{1,1}^g + h_{1,3}^g)$, $N^g(\psi^+) = 4h_{1,2}^g$ and $N^g(\varphi^+) = \chi^g + 4h_{1,2}^g$.

3.3 Heterotic

In heterotic compactification to $D=2$ there is a potential B -field tadpole arising from the $D=10$ action term that cancels the anomaly, namely $\int B \wedge X_8$ [2]. In the notation of [30], the 8-form X_8 is

$$8X_8 = \frac{1}{3}\text{Tr}_A F^4 - \frac{1}{900}(\text{Tr}_A F^2)^2 - \frac{1}{30}\text{Tr}_A F^2 \text{tr} R^2 + \text{tr} R^4 + \frac{1}{4}(\text{tr} R^2)^2 . \quad (25)$$

With a convenient normalization the coefficient of the tadpole is

$$c = \frac{1}{48(2\pi)^4} \int_Y X_8 . \quad (26)$$

Clearly, c could vanish when the gauge fields have a precise background.

An alternative way of determining c is to calculate the anomaly polynomial of the $D=2$ theory obtained upon compactification. This anomaly must be exactly of the form

$$\mathcal{A} = c (\text{tr} R^2 - v_a \text{tr} F_a^2) , \quad (27)$$

where, using conventions specified below, $v_a = 1$ for all group factors. The point is that this anomaly would be canceled through the Green-Schwarz mechanism precisely by a term $c \int B$ in the $D=2$ action. In the case of the standard embedding we will compute c in both ways and show agreement. To this end we first determine the massless fields upon reduction.

In the heterotic string there is one $D=10$ gravitino, chosen in the $\mathbf{16}_c$, that produces \mathcal{N} positive chirality gravitini in $D=2$. Hence, the resulting theory has $(\mathcal{N}, 0)$ supersymmetry. The dilatino in the $\mathbf{16}_s$ gives \mathcal{N} negative chirality M-W spinors that also belong in the gravity multiplet. The gravitino components with internal indices produce M-W spinors, $2(h_{1,1} + h_{1,3})$ of negative and $2h_{1,2}$ of positive chirality. From the metric and the antisymmetric tensor there are $2(h_{1,1} + h_{1,3})$ scalars.

We next perform the reduction of the $D=10$ gauge multiplet with group \mathcal{G} equal to $E_8 \times E_8$ or SO_{32} . We will assume that $h_{0,2} = 0$ so that the holonomy is exactly SU_4 . We can then realize the standard embedding, i.e. choose a background for the gauge connection equal to the spin connection. This background breaks \mathcal{G} to the commutant G that gives a maximal subgroup $G \times SU_4 \subset \mathcal{G}$. Then, the standard embedding gives group $SO_{10} \times E_8$ or $SO_{24} \times U_1$.

To determine the massless fermions in various representations we look for zero modes of the $D=10$ gaugini λ_α that are also M-W in the **16**_c. The standard embedding implies that SU_4 gauge transformations are identified with SU_4 holonomy transformations. This means for instance that the internal Dirac operator acting on gaugini that are SU_4 singlets is just like the internal Dirac operator acting on neutral spinors. We already know that there are two zero modes η_+ and η_- in this case. Therefore, SU_4 singlet gaugini just give rise to two positive chirality gaugini of the unbroken group G in $D=2$.

To study SU_4 charged gaugini we need to decompose the \mathcal{G} adjoint under $G \times SU_4$. For $SO_{10} \times SU_4 \subset E_8$ we have

$$\mathbf{248} = (\mathbf{45}, \mathbf{1}) \oplus (\mathbf{16}, \mathbf{4}) \oplus (\overline{\mathbf{16}}, \overline{\mathbf{4}}) \oplus (\mathbf{10}, \mathbf{6}) \oplus (\mathbf{1}, \mathbf{15}) . \quad (28)$$

We see that there are gaugini, transforming in the SO_{10} adjoint, that are SU_4 singlets and that were already discussed in the previous paragraph. There are also components, denoted λ_i , that transform in the **16** of SO_{10} and the **4** of SU_4 . Due to the SU_4 index i , zero modes of λ_i are just like zero modes of the gravitino ψ_i , and similarly for the $\lambda_{\bar{i}}$ in the $\overline{\mathbf{4}}$. Hence, there are $(h_{1,1} + h_{1,3})$ negative chirality and $h_{1,2}$ positive chirality spinors transforming in **16** + $\overline{\mathbf{16}}$ of the unbroken SO_{10} . Since the **6** of SU_4 is the antisymmetric product of two fundamentals, we can argue that for the components transforming in the **10** of SO_{10} and the **6** of SU_4 the internal spinors in the decomposition of type (15) have two antisymmetric holomorphic indices so that the zero modes correspond to $(2, q)$ harmonic forms. This then gives $2h_{1,2}$ negative chirality and $h_{2,2}$ positive chirality spinors transforming in **10**. We will see that this result is consistent with anomaly factorization. We can also check it explicitly in orbifold examples. For the components transforming as **15** of SU_4 we will just assume that they give a net number N_{0G} of positive chirality spinors singlets of SO_{10} .

To compute the anomaly polynomial we need the contribution of fermions transforming in a representation \mathcal{R} of the unbroken gauge group. For positive chirality this is

$$I_{1/2}^+ = \frac{\dim \mathcal{R}}{48} \text{tr } R^2 - \frac{1}{2} \text{Tr}_{\mathcal{R}} F^2 . \quad (29)$$

In general we can write $\text{Tr}_{\mathcal{R}} F^2 = T(\mathcal{R}) \text{tr } F^2$. We use conventions such that for E_8 , $T(\mathbf{496}) = 30$, for SU_N , $T(\mathcal{R}) = \frac{1}{2}, \frac{N-2}{2}, N$, for the fundamental, 2-index antisymmetric

and adjoint, for SO_{2N} , $T(\mathcal{R}) = 1, 2^{N-4}, (2N+2), (2N-2)$, for the vector, spinor, 2-index symmetric and adjoint. The full anomaly polynomial takes the form (27), only the overall constant c is model dependent. In compactifications of the $E_8 \times E_8$ heterotic on a CY₄ with standard embedding, necessarily $c = 30$ since the unbroken group includes the hidden E_8 . The same coefficient for the observable SO_{10} follows, using (1) for $h_{2,2}$, provided that the numbers of spinors transforming as **10** are exactly as claimed before. Requiring that the gravitational anomaly also appears with $c = 30$ fixes the net number of singlet spinors arising from the $D=10$ gaugini. We find

$$N_{0G} = 510 - \chi . \quad (30)$$

Recall that there are spinors arising from the gravitino that are obviously gauge singlets. Taking these into account, the total net number of positive chirality singlet spinors is $N_{0T} = 526 - 4\chi/3$. This result can be verified in explicit orbifold examples.

To determine the spinors arising from gaugini in the SO_{32} heterotic compactified on a CY₄ with standard embedding, we need the decomposition of the adjoint **496** under $SO_{24} \times U_1 \times SU_4$. This is

$$\mathbf{496} = (\mathbf{276}, 0, \mathbf{1}) \oplus (\mathbf{1}, 0, \mathbf{1}) \oplus (\mathbf{24}, q, \mathbf{4}) \oplus (\mathbf{24}, -q, \bar{\mathbf{4}}) \oplus (\mathbf{1}, 2q, \mathbf{6}) \oplus (\mathbf{1}, -2q, \mathbf{6}) \oplus (\mathbf{1}, 0, \mathbf{15}) , \quad (31)$$

where $q = \frac{1}{2\sqrt{2}}$ in order that the U_1 generator Q has the correct normalization $\text{Tr } Q^2 = 30$. According to our previous discussion the SU_4 background breaks SO_{32} to $SO_{24} \times U_1$ with the following charged spinor content: 2 positive chirality gaugini transforming in the adjoint, $(h_{1,1} + h_{1,3})$ negative chirality and $h_{1,2}$ positive chirality spinors transforming in $(\mathbf{24}, q) + (\mathbf{24}, -q)$, as well as $2h_{1,2}$ negative chirality and $h_{2,2}$ positive chirality spinors transforming in $(\mathbf{1}, 2q) + (\mathbf{1}, -2q)$. Moreover, there must be a net number N_{0G} of positive chirality spinor singlets $(\mathbf{1}, 0)$. The SO_{24} and U_1 gauge pieces of the full anomaly polynomial are easy to compute. Using (2) and (1) both give $c = 30 - \frac{\chi}{6}$. The gravitational piece appears with this same c provided that N_{0G} is given by (30), as expected since this value is a property of the four-fold.

It remains to analyze the zero modes of the $D=10$ gauge vectors. Besides the (non-dynamical) vectors partners of the $D=2$ gaugini, by supersymmetry we expect to obtain scalars to pair into chiral multiplets with the negative chirality spinors. For instance, in

the $E_8 \times E_8$ heterotic there must be $(h_{1,1} + h_{1,3})$ real scalars transforming in $\mathbf{16} + \overline{\mathbf{16}}$ and $2h_{1,2}$ in $\mathbf{10}$. It is useful to organize the fields into multiplets of $(2,0)$ supersymmetry that can be obtained decomposing those of $(2,2)$. The gauge multiplet contains gauge vectors and two positive chirality M-W spinors. The chiral multiplet, denoted generically Φ_2^- , contains one complex scalar and one negative chirality Weyl spinor. The so called Fermi multiplet [35] contains one positive chirality Weyl spinor, denoted generically Ψ^+ . Then, for example, the $(2,0)$ theory arising upon compactification of the $E_8 \times E_8$ heterotic on a CY₄ has the following multiplets

$$\Phi_2^-[(h_{1,1} + h_{1,3})(\mathbf{16} + \mathbf{1}) + h_{1,2}\mathbf{10}] + \Psi^+[\frac{h_{2,2}}{2}\mathbf{10} + \frac{N_{0G}}{2}\mathbf{1}] , \quad (32)$$

where we have not included the $S0_{10} \times E_8$ gauge multiplets. All fields in (32) are E_8 singlets. For the SO_{32} heterotic the observable group is $S0_{24} \times U_1$ and in (32) we just need to replace $\mathbf{16}$ by $(\mathbf{24}, q)$, $\mathbf{10}$ by $(\mathbf{1}, 2q)$ and $\mathbf{1}$ by $(\mathbf{1}, 0)$. These results are reproduced in orbifold compactifications discussed shortly.

Now that we have computed the tadpole coefficient from the anomaly we wish to show that the same result follows from (26). The task is basically to determine the adjoint traces $\text{Tr}_A F^2$ and $\text{Tr}_A F^4$ in the standard embedding. To this end we use the adjoint decompositions, (28) or (31), our SU_4 conventions for $\text{Tr}_R F^2$ and the relations (see the appendix of [36])

$$\text{Tr}_4 F^4 = \frac{3}{16}(\text{tr} F^2)^2 - \frac{1}{4}\text{tr} F^4 \quad ; \quad \text{Tr}_6 F^4 = \text{tr} F^4 \quad ; \quad \text{Tr}_{15} F^4 = 3(\text{tr} F^2)^2 - 2\text{tr} F^4 .$$

Furthermore, since $\text{tr} R^2$ is evaluated in the $\mathbf{8}_v = \mathbf{4} + \bar{\mathbf{4}}$, we find that in the standard embedding $\text{tr} F^2 = \text{tr} R^2$ while $\text{tr} F^4 = \frac{3}{4}(\text{tr} R^2)^2 - 2\text{tr} R^4$. Collecting all intermediate results we arrive at

$$X_8^{E_8 \times E_8} = \frac{1}{8}\text{tr} R^4 + \frac{5}{32}(\text{tr} R^2)^2 \quad ; \quad X_8^{SO_{32}} = \frac{9}{8}\text{tr} R^4 - \frac{3}{32}(\text{tr} R^2)^2 . \quad (33)$$

Using (4) to perform the integration we readily find $c(E_8 \times E_8) = 30$ and $c(SO_{32}) = 30 - \frac{\chi}{6}$, in accordance with the anomaly computation. Thus, with standard embedding only the perturbative SO_{32} heterotic could be tadpole-free provided the four-fold has $\chi = 180$. Such four-folds do exist [37]. For instance, there are 28 of them among the transversal hypersurfaces in weighted projective space. The example of lowest degree and reflexive weights is a hypersurface in $\mathbb{P}_5^{(9,9,11,18,22,30)}$ [99] having $h_{1,1} = 27$, $h_{1,2} = 60$ and $h_{1,3} = 55$.

It is also interesting to discuss four-folds with $h_{0,2} \neq 0$. For example, for $K3 \times K3$ with $h_{0,2} = 2$, the holonomy group is $SU_2 \times SU_2$. The standard embedding then gives group $SO_{12} \times E_8$ or $SO_{24} \times U_1^2$. In the $E_8 \times E_8$ heterotic matter can be neutral or transform in the spinorial **32** or **32'** or vector **12** of the commutant SO_{12} . Based on the branching of the E_8 adjoint representation under $SO_{12} \times SU_2 \times SU_2$ we find $\frac{1}{2}(h_{1,1} + h_{1,3})$ negative chirality and $\frac{1}{2}h_{1,2}$ positive chirality M-W spinors transforming in **32 + 32'**, as well as $2h_{1,2}$ negative chirality and $(h_{2,2} - 4)$ positive chirality spinors transforming in **12**. From anomaly factorization we find the total net number of positive chirality singlet spinors (including modulini) to be $N_{0T} = 1016 - 8\chi/3$. These same results are obtained in orbifold realizations of $K3 \times K3$ as well as in other $h_{0,2} = 2$, \mathbb{Z}_N or $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds, with or without discrete torsion. The SO_{32} heterotic is just as simple to work out. In general, in orbifolds with $h_{0,2} \neq 0$ and standard embedding the tadpole coefficients turn out to be $c(E_8 \times E_8) = 15(2 + h_{0,2})$ and $c(SO_{32}) = 15(2 + h_{0,2}) - \frac{\chi}{6}$. Instead of having to choose an internal space with a precise χ an obvious alternative to cancel the tadpole is to make a different embedding and this can be most easily carried out in orbifold compactifications.

Compactification of heterotic strings on T^8/P orbifolds, in particular computation of the massless spectrum, follows as usual [33, 32] and taking into account the slight modification explained at the end of section 3.2. To begin we need to specify the embedding in the gauge degrees of freedom. We use the bosonic formulation and realize a \mathbb{Z}_N rotation (6) by a shift vector V such that $NV \in \Gamma$, where Γ is either the $E_8 \times E_8$ or the $Spin(32)/\mathbb{Z}_2$ lattice. Modular invariance of the partition function requires

$$N(V^2 - v^2) = 0 \bmod 2 . \quad (34)$$

The standard embedding $V = (v_1, v_2, v_3, v_4, 0, \dots, 0)$ trivially satisfies (34) but there are many other solutions.

The orbifolds with $h_{0,2} = 0$ are presumably singular limits of CY₄'s. The simplest example to compare with smooth compactification is the \mathbb{Z}_4 in Table 1 with

$$\begin{array}{cccccc} g & h_{1,1}^g & h_{1,2}^g & h_{1,3}^g & h_{2,2}^g & \chi^g \\ \hline \theta^0 & 16 & 0 & 0 & 36 & 72 \\ \theta + \theta^3 & 16 & 0 & 0 & 0 & 32 \\ \theta^2 & 0 & 0 & 0 & 136 & 136 \end{array} . \quad (35)$$

The shift V for the standard embedding breaks SO_{32} to $SO_{24} \times U_1 \times SU_4$. We find the following (2,0) massless matter multiplets:

$$\begin{aligned}\theta^0 : \quad & 3\Psi^+(\mathbf{1}, 2q, \mathbf{6}) + 4\Phi_2^-[(\mathbf{24}, q, \mathbf{4}) + 4(\mathbf{1}, 0, \mathbf{1})] \\ \theta + \theta^3 : \quad & 16\Phi_2^-[(\mathbf{24}, q, \mathbf{1}) + 4(\mathbf{1}, 0, \mathbf{4})] \\ \theta^2 : \quad & \Psi^+[\mathbf{68}(\mathbf{1}, 2q, \mathbf{1}) + 60(\mathbf{1}, 0, \mathbf{6})]\end{aligned}, \quad (36)$$

where $q = 1/2\sqrt{2}$. Using the data in (35) we can check that in the twisted sectors the number of $SO_{24} \times U_1$ charged multiplets agrees with the analysis for smooth CY₄'s, c.f. (32). Agreement in the untwisted sector requires counting SU_4 dimensionality as multiplicity, i.e. assuming that SU_4 is completely broken. Among the massless states there are candidate Higgs fields, namely the scalars that are inert under $SO_{24} \times U_1$ and sit at each fixed point in the $\theta + \theta^3$ sectors. These orbifold states are indeed of blowing-up type since they have left-moving oscillators acting on the twisted vacuum [38]. We also find that the total net number of positive chirality spinors is $N_{0T} = 526 - 4\chi/3$ provided that we include the SU_4 gaugini. It is straightforward to do the standard embedding for other CY₄ orbifolds of the $E_8 \times E_8$ or the SO_{32} heterotic string. Some examples were studied in [8]. In all cases there are scalars that can break the observable group to SO_{10} or $SO_{24} \times U_1$.

We now wish to present an example with vanishing tadpole in the SO_{32} heterotic. For the \mathbb{Z}_3 of Table 1 we take the non-standard embedding

$$V = \frac{1}{3}(1, \dots, 1, 0, 0, 0, 0, 0, 0, 0), \quad (37)$$

which leaves $SU_{10} \times SO_{12} \times U_1$ unbroken. The massless spectrum is

$$\begin{aligned}\theta^0 : \quad & (\Phi_6^- + \Psi^+)[(\mathbf{10}, \mathbf{12}, \tfrac{1}{2\sqrt{5}}) + (\mathbf{45}, \mathbf{1}, -\tfrac{1}{\sqrt{5}}) + 4(\mathbf{1}, \mathbf{1}, 0)] \\ \theta + \theta^2 : \quad & 81\Psi^+(\mathbf{1}, \mathbf{1}, \tfrac{\sqrt{5}}{3}),\end{aligned}, \quad (38)$$

where Φ_6^- stands for a (6,0) chiral multiplet that includes four complex scalars and four negative chirality Weyl spinors. It is easy to check the vanishing of the full anomaly polynomial. Another distinctive feature is the absence of non-Abelian charged matter in the twisted sectors, an important fact in identifying this orbifold as dual to an orientifold studied in the next section.

Taking hints from the orientifolds of next section we have found other SO_{32} non-standard embeddings leading to models with $c = 0$. There is one such model for each \mathbb{Z}_N with N odd. The corresponding shifts and unbroken group are

N	V	G	
5	$(\frac{1}{5}^6, \frac{2}{5}^6, 0^4)$	$U_6^2 \times SO_8$,
9	$(\frac{1}{9}^4, \frac{2}{9}^4, \frac{4}{9}^4, \frac{3}{9}^2, 0^2)$	$U_4^3 \times U_2 \times SO_4$	
15	$(\frac{1}{15}^2, \frac{2}{15}^2, \frac{4}{15}^2, \frac{8}{15}^2, \frac{3}{15}^2 \frac{6}{15}^2, \frac{5}{15}^2, 0^2)$	$U_2^7 \times SO_4$	

where the notation, e.g. $\frac{1}{5}^6$ means $\frac{1}{5}$ repeated six times. The massless spectra are straightforward to compute but too cumbersome to display. In all cases we find that the full anomaly polynomial, including U_1 terms, does vanish.

3.4 Type I

It is natural to study type I compactifications that will give $(\mathcal{N}, 0)$ theories with a charged sector arising from Dp-branes. These vacua can be described as $D=2$ type IIB orientifolds. These orientifolds must be free of gravitational and non-Abelian anomalies. There could only be U_1 anomalies that can be canceled by couplings of RR scalars to the U_1 field strength. We will show explicit examples having these properties.

We follow the construction used in [39, 40] for $D=6$ and in [41] for $D=4$. The orientifold group is $G_1 + \Omega G_2$, we focus mostly on $G_1 = G_2 = \mathbb{Z}_N$. The closed string states are those of type IIB on T^8/\mathbb{Z}_N invariant under Ω . The open string states depend on the matrices $\gamma_{g,p}$ that realize the \mathbb{Z}_N action on the Chan-Paton factors. These matrices are constrained by the orientifold group structure and tadpole cancellation.

Tadpoles are particularly easy to compute for N odd. In this case cancellation requires D9-branes only, $\gamma_{\Omega,9}^T = \gamma_{\Omega,9}$ and

$$\text{Tr } \gamma_{2k,9} = 32 \prod_{j=1}^4 \cos k\pi v_j \quad ; \quad k = 0, 1, \dots, N-1 . \quad (40)$$

It is also consistent to take $\gamma_{1,9}^N = \mathbb{1}$. As a simple example consider the \mathbb{Z}_3 of Table 1. The solution to (40) is

$$\gamma_{1,9} = \text{diag}(e^{2i\pi/3}\mathbb{1}_{10}, e^{-2i\pi/3}\mathbb{1}_{10}, \mathbb{1}_{12}) . \quad (41)$$

The resulting gauge group is $SU_{10} \times SO_{12} \times U_1$. The open string spectrum includes exactly the same charged fields appearing in the untwisted sector of the heterotic orbifold (38). This is not surprising since in the auxiliary shift formalism [41], (41) is equivalent to (37). The closed string spectrum is

$$\begin{aligned} \theta^0 : & \quad 4(\Phi_6^- + \Psi^+)(\mathbf{1}, \mathbf{1}, 0) \\ \theta + \theta^2 : & \quad 81\phi^+(\mathbf{1}, \mathbf{1}, 0) \end{aligned} . \quad (42)$$

In the untwisted sector there appear precisely the neutral multiplets needed to complete the heterotic untwisted sector. In the twisted sectors instead of the positive chirality spinors found in the heterotic, there appear positive chirality scalars. Upon fermionization the number of fields is the same and the total gravitational, as well as the gauge non-Abelian anomalies are still identically zero. The orientifold U_1 is anomalous because the ϕ^+ 's are not charged. However, based on general arguments the anomaly can be compensated by anomalous U_1 transformations of the ϕ^+ that are RR scalars [42].

In $D=2$ the type I and the heterotic dilaton are identical [43] so that examples of exact weak/weak duals in which the perturbative spectra coincide are expected. This can occur in $D=4$ as well [43, 44]. In the other $D=2$ odd \mathbb{Z}_N orientifolds the 99 sector fully matches the charged untwisted multiplets of a perturbative SO_{32} heterotic with embedding given by the auxiliary shift. The untwisted closed states complete the heterotic untwisted sector. In \mathbb{Z}_5 and \mathbb{Z}_{15} the heterotic twisted states are all non-Abelian singlets and match the twisted closed states up to fermionization and U_1 charges. In \mathbb{Z}_9 the heterotic θ^3, θ^6 sectors include some non-Abelian charged fields that could be Higgsed away as in [44]. The gauge group is of the form $\prod_\alpha U_n^\alpha \times SO_{2\ell}$, where $\alpha = 1, \dots, \frac{(N-1)}{2}$. The auxiliary shift V has components V^α repeated n_α times plus ℓ zeroes, see (37) and (39). The U_1^α in each U_n^α is anomalous but the anomaly can be compensated by transformation of RR scalars.

We can adapt the analysis of [42] to verify cancellation of the 1-loop anomaly that in $D=2$ comes from the vacuum polarization diagram. In the closed string channel the relevant counter diagram is the annulus with RR fields propagating along and one U_1^α coupled at each boundary. The contribution to the anomaly coming from this graph is

proportional to

$$A_\alpha = \frac{n_\alpha}{N} \sum_{k=1}^{N-1} C_k(v) \sin^2 2\pi k V^\alpha , \quad (43)$$

where the coefficients $C_k(v)$ are

$$C_k(v) = \prod_{i=1}^4 2 \sin k\pi v_i . \quad (44)$$

In all cases we find that A_α equals the coefficient of the 1-loop anomaly, namely $-\frac{1}{2}\text{Tr } Q_\alpha^2$ with normalization such that Q_α charges are multiples of $1/\sqrt{2n_\alpha}$. Mixed $U_1^\alpha U_1^\beta$ anomalies cancel in similar fashion. Cancellation of U_1 anomalies in models with D1-branes at $\mathbb{C}^4/\mathbb{Z}_N$ singularities through the same mechanism was discussed in [45].

In addition to D9-branes, even order orientifolds require D5-branes and/or D1-branes. A simple example with only D5-branes is the \mathbb{Z}_4 generated by $\frac{1}{4}(2, -2, 1, -1)$. Tadpole cancellation implies 99 group U_8^2 . With all D5-branes at the origin the 55 group is the same. The matter content is somehow similar to that in the $D=6$, \mathbb{Z}_4 orientifold [40]. Examples with D5 and D1-branes were analyzed in [18].

We now wish to discuss an orientifold with D1-branes and to this end consider the \mathbb{Z}_2 of Table 1 whose generator will be denoted R . This example was first studied in [18]. The structure of tadpoles is very similar to that in the GP orientifold [39]. There are the usual tadpoles proportional to $V_2 V_8$ that require 32 D9-branes (V_2 and V_8 are respectively the regularized space-time volume and the torus volume). There are also tadpoles proportional to V_2/V_8 that require 32 1-branes. Furthermore,

$$\gamma_{\Omega,9}^T = \gamma_{\Omega,9} \quad ; \quad \gamma_{\Omega R,1}^T = \gamma_{\Omega R,1} \quad (45)$$

$$\gamma_{\Omega,1}^T = \gamma_{\Omega,1} \quad ; \quad \gamma_{\Omega R,9}^T = \gamma_{\Omega R,9} , \quad (46)$$

where the first line is needed for tadpole cancellation and the second follows because the GP action is such that $\Omega^2 = \mathbb{1}$ also on 19-states. Finally, there are the twisted tadpoles proportional to V_2 and to

$$\sum_{I=1}^{256} (\text{Tr } \gamma_{R,9} + 16 \text{Tr } \gamma_{R,1,I})^2 . \quad (47)$$

Cancellation of (47) implies $\text{Tr } \gamma_{R,9} = \text{Tr } \gamma_{R,1,I} = 0$. I runs over the fixed points of R .

Without loss of generality we can take $\gamma_{\Omega,9} = \gamma_{\Omega,1} = \mathbb{1}$. The algebra then implies that $\gamma_{R,9} = \gamma_{\Omega R,9}$ and $\gamma_{R,1} = \gamma_{\Omega R,1}$ so that the matrices $\gamma_{R,p}$ are symmetric and traceless. We

can choose

$$\gamma_{R,9} = \text{diag}(\mathbb{1}_{16}, -\mathbb{1}_{16}) \quad ; \quad \gamma_{R,1,I} = \text{diag}(\mathbb{1}_{m_I}, -\mathbb{1}_{m_I}) \quad (48)$$

where $2m_I$ is the number of 1-branes at fixed point I .

The gauge group for 99 strings is $SO_{16} \times SO_{16}$ and the matter content is an (8,0) chiral multiplet Φ_8^- transforming in **(16, 16)** (Φ_8^- contains 4 complex scalars and 4 negative chirality Weyl spinors). For $1_J 1_I$ strings the gauge group is $SO_{m_I} \times SO_{m_I}$ and matter is a Φ_8^- in $(\mathbf{m}_I, \mathbf{m}_I)$. There can also be m_J D1-branes sitting at a non-fixed point J (and thus the same number at the \mathbb{Z}_2 image). Then, $\sum_I m_I + \sum_J m_J = 16$. The gauge group for $1_J 1_J$ strings is SO_{m_J} since the R projection only exchanges J and its image, and the Ω projection just implies anti-symmetric Chan-Paton factor. There is also a Φ_8^- transforming in $\frac{\mathbf{m}_J(\mathbf{m}_J+1)}{2}$, the Ω projection implies symmetric Chan-Paton because of the extra sign due to DD boundary conditions.

For 19 strings inspection of the cylinder partition function reveals no massless states in the NS sector and only one positive chirality M-W spinor in the R sector. This spinor is invariant under the \mathbb{Z}_2 rotation so the R projection implies invariant Chan-Paton factor. The representations are then **(16, 1; \mathbf{m}_I , 1)** plus **(1, 16; $\mathbf{1}, \mathbf{m}_I$)** for 91_I strings and for 91_J , **(16, 1; \mathbf{m}_J)** plus **(1, 16; \mathbf{m}_J)**.

Finally, the closed strings provide the (8,0) gravity multiplet and the moduli, i.e. eight singlet chiral multiplets, from the untwisted sector. The Ω projection removes all states from the twisted sector. Given the full gauge and matter content it is easy to check that the gravitational and gauge anomalies do vanish.

Unlike the GP model, now there is no way to cancel tadpoles locally so we do not expect to find a weak heterotic dual. A very simple configuration, with group just $SO_{16} \times SO_{16}$ from 99 strings, has 16 wandering D1-branes. Matter is comprised by one chiral multiplet in **(16, 16)** from 99 strings, sixteen singlet chiral multiplets from 11 strings, sixteen positive chirality M-W spinors in **(16, 1)** plus **(1, 16)** from 91_J , and the moduli from the closed untwisted strings.

It is rather easy to find a V giving $SO_{16} \times SO_{16}$ upon heterotic compactification on T^8/\mathbb{Z}_2 . In both heterotics V is such that there is no massless matter in the twisted sector. In the untwisted sector we find the moduli together with one charged chiral multiplet, transforming as **(16, 16)** and as **(128, 1) + (1, 128)** in the SO_{32} and the $E_8 \times E_8$ heterotic

respectively. The two heterotics have total anomaly polynomial of the form (27) with $c = -8$. It is tempting to suppose that in the SO_{32} the tadpole can be canceled by adding sixteen 1-branes, dual to the wandering D1-branes in the orientifold. In the $E_8 \times E_8$ the tadpole could be canceled by adding wrapped M2-branes.

When $c < 0$, adding strings dual to wandering D1-branes can offset the tadpole because each brane by itself has an anomaly polynomial of the form (27) with coefficient $c_1 = \frac{1}{2}$. Indeed, recall that from 11 strings the degrees of freedom are eight real scalars and eight M-W spinors of negative chirality whereas from 19 strings there is one positive chirality M-W spinor originally transforming in the vector of SO_{32} .

4 Conclusions

Consistent compactifications to $D=2$ do belong in the full landscape of string vacua and deserve further study. Constructing T^8/P orbifolds and using them to compactify strings is a natural project that, to our knowledge, had not been previously carried out systematically. In this paper we have partially classified such orbifolds with $P \subset SU_4$. We have also explained how to adapt the usual rules to compute the orbifold spectrum of states to the $D=2$ case that requires care for chiral theories.

We have noticed that the B -field tadpole generically present in heterotic compactifications is most easily computed as the coefficient of the anomaly polynomial. In the perturbative heterotic string this coefficient follows directly from the massless spectrum. This gives a simple prescription to search for perturbative tadpole-free models. We have discovered a few examples in the SO_{32} heterotic. Those with standard embedding require an internal space with $\chi = 90(2 + h_{0,2})$. We have not found this Euler number in orbifolds but rather in Calabi-Yau four-folds. The tadpole-free models with non-standard embedding are connected to type IIB \mathbb{Z}_{odd} orientifolds. In fact, analyzing orientifolds in which the anomaly is always absent suggests possible ways to cancel the heterotic tadpole via non-perturbative effects, for instance turning on strings dual to wandering D1-branes.

Among possible applications of our results we can envisage the study of the effective $D=2$ theories. For example, as in [14, 16, 46] one could consider type IIA compactifications on T^8/P orbifolds. Finally, let us remark that M-Theory and F-theory compactifications

on these orbifolds can be explored as well.

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P	Generator	$h_{0,2}$	$h_{1,1}$	$h_{1,2}$	$h_{1,3}$	$h_{2,2}$	$\chi/24$
\mathbb{Z}_2	$(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$	6	16	0	16	292	16
\mathbb{Z}_3	$(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$	4	8	4	8	180	9
$\mathbb{Z}_N, N = 4, 6$	$(\frac{1}{N}, -\frac{1}{N}, \frac{1}{N}, -\frac{1}{N})$	4	8	0	8	188	10
$\mathbb{Z}_N, N = 4, 6$	$(\frac{1}{2}, -\frac{1}{2}, \frac{1}{N}, -\frac{1}{N})$	2	16	16	16	180	8
\mathbb{Z}_5	$(\frac{1}{5}, \frac{3}{5}, -\frac{1}{5}, -\frac{3}{5})$	2	4	4	4	108	5
\mathbb{Z}_8	$(\frac{1}{8}, \frac{3}{8}, -\frac{1}{8}, -\frac{3}{8})$	2	4	0	4	116	6
\mathbb{Z}_4	$(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{3}{4})$	0	32	0	0	172	10
\mathbb{Z}_6	$(\frac{1}{6}, \frac{1}{6}, \frac{2}{6}, -\frac{4}{6})$	0	23	12	5	132	6
\mathbb{Z}_9	$(\frac{1}{9}, \frac{4}{9}, -\frac{2}{9}, -\frac{3}{9})$	0	15	11	0	82	3
\mathbb{Z}_{15}	$(\frac{1}{15}, \frac{2}{15}, \frac{4}{15}, -\frac{7}{15})$	0	8	0	0	76	4

Table 1: T^8/\mathbb{Z}_N

P	Generators	$h_{0,2}$	$h_{1,1}$	$h_{1,2}$	$h_{1,3}$	$h_{2,2}$	$\chi/24$
$\mathbb{Z}_N \times \mathbb{Z}_M$ $N, M = 2, 3, 4, 6$	$(\frac{1}{N}, -\frac{1}{N}, 0, 0)$ $(0, 0, \frac{1}{M}, -\frac{1}{M})$	2	40	0	40	404	24
$\mathbb{Z}_2 \times \mathbb{Z}_3$	$(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$ $(0, 0, \frac{1}{3}, -\frac{1}{3})$	2	16	16	16	180	8
$\mathbb{Z}_2 \times \mathbb{Z}_3$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$ $(0, \frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$	0	32	21	5	150	6
$\mathbb{Z}_2 \times \mathbb{Z}_3$	$(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$ $(0, \frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$	0	28	13	1	134	6
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$ $(0, \frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$	0	54	16	2	236	12
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$ $(0, \frac{1}{4}, \frac{1}{4}, -\frac{1}{2})$	0	64	0	8	332	20
$\mathbb{Z}_2 \times \mathbb{Z}_7$	$(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$ $(0, \frac{1}{7}, \frac{2}{7}, -\frac{3}{7})$	0	16	9	1	94	4

Table 2: $T^8/\mathbb{Z}_N \times \mathbb{Z}_M$

P	Generators	$h_{0,2}$	$h_{1,1}$	$h_{1,2}$	$h_{1,3}$	$h_{2,2}$	$\chi/24$
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$ $(0, 0, \frac{1}{2}, -\frac{1}{2})$ $(0, \frac{1}{2}, -\frac{1}{2}, 0)$	0	100	0	4	460	28
$\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_2$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$ $(0, 0, \frac{1}{3}, -\frac{1}{3})$ $(0, \frac{1}{2}, -\frac{1}{2}, 0)$	0	72	8	8	348	20
$\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_2$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$ $(0, 0, \frac{1}{4}, -\frac{1}{4})$ $(0, \frac{1}{2}, -\frac{1}{2}, 0)$	0	118	0	2	524	32
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$ $(0, 0, \frac{1}{2}, -\frac{1}{2})$ $(0, \frac{1}{3}, -\frac{1}{3}, 0)$	0	44	16	12	236	12
$\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_3$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$ $(0, 0, \frac{1}{3}, -\frac{1}{3})$ $(0, \frac{1}{3}, -\frac{1}{3}, 0)$	0	96	21	5	406	22
$\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3$	$(\frac{1}{3}, -\frac{1}{3}, 0, 0)$ $(0, 0, \frac{1}{3}, -\frac{1}{3})$ $(0, \frac{1}{3}, -\frac{1}{3}, 0)$	0	220	0	0	924	57

Table 3: $T^8/\mathbb{Z}_N \times \mathbb{Z}_M \times \mathbb{Z}_K$

P	Generators	Phases	$h_{0,2}$	$h_{1,1}$	$h_{1,2}$	$h_{1,3}$	$h_{2,2}$	$\chi/24$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$	$\epsilon_1 = -1$	2	8	64	8	20	-8
	$(0, 0, \frac{1}{2}, -\frac{1}{2})$							
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(\frac{1}{3}, -\frac{1}{3}, 0, 0)$	$\epsilon_1 = e^{2i\pi/3}$	2	4	36	4	44	-3
	$(0, 0, \frac{1}{3}, -\frac{1}{3})$							
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(\frac{1}{4}, -\frac{1}{4}, 0, 0)$	$\epsilon_1 = -1$	2	24	0	24	276	16
	$(0, 0, \frac{1}{4}, -\frac{1}{4})$	$\epsilon_1 = i$	2	4	16	4	84	2
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$	$\epsilon_1 = 1, \epsilon_3 = -1$						
	$(0, 0, \frac{1}{2}, -\frac{1}{2})$	$\epsilon_2 = 1$	0	20	64	20	76	-4
	$(0, \frac{1}{2}, -\frac{1}{2}, 0)$	$\epsilon_2 = -1$	0	4	0	100	460	28
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$	$(\frac{1}{2}, -\frac{1}{2}, 0, 0)$	$\epsilon_1 = -1$	0	20	48	4	44	-4
	$(0, 0, \frac{1}{2}, -\frac{1}{2})$							
	$(0, \frac{1}{3}, -\frac{1}{3}, 0)$							
$\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3$	$(\frac{1}{3}, -\frac{1}{3}, 0, 0)$	$\epsilon_1 = 1, \epsilon_3 = e^{2i\pi/3}$						
	$(0, 0, \frac{1}{3}, -\frac{1}{3})$	$\epsilon_2 = 1$	0	22	63	9	42	-6
	$(0, \frac{1}{3}, -\frac{1}{3}, 0)$	$\epsilon_2 = e^{2i\pi/3}$	0	4	0	36	204	12

Table 4: Examples with discrete torsion